ABSTRACT

A geometrical approach to two-dimensional Conformal Field Theory

This thesis is organized in the following way. In Chapter 2 we will give a brief introduction to conformal field theory along the lines of standard quantum field theory, without any claims to originality. We introduce the important concepts of the stress-energy tensor, the Virasoro algebra, and primary fields. The general principles are demonstrated by fermionic and bosonic free field theories. This also allows us to discuss some general aspects of moduli spaces of CFT's. In particular, we describe in some detail the space of inequivalent toroidal compactifications, giving examples of the quantum equivalences that we already mentioned. In Chapter 3 we will reconsider general quantum field theory from a more geometrical point of view, along the lines of the so-called operator formalism. Crucial to this approach will be the consideration of topology changing amplitudes. After a simple application to 2d topological theories, we proceed to give our second introduction to CFT, stressing the geometry behind it. In Chapter 4 the so-called rational conformal field theories are our object of study. These special CFT's have extended symmetries with only a finite number of representations. If an interpretation as non-linear sigma model exists, this extra symmetry can be seen as a kind of resonance effect due to the commensurability of the size of the string and the target space-time. The structure of rational CFT's is extremely rigid, and one of our results will be that the operator content of these models is—up to some discrete choices—completely determined by the symmetry algebra. The study of rational models is in its rigidity very analogous to finite group theory. In Chapter 5 this analogy is further pursued and substantiated. We will show how one can construct from general grounds rational conformal field theories from finite groups. These models are abstract versions of non-linear o-models describing string propagation on 'orbifolds.' An orbifold is a singular manifold obtained as the quotient of a smooth manifold by a discrete group. In Chapter 6 our considerations will be of a somewhat complementary nature. We will investigate models with central charge c = 1 by deformation techniques. The central charge is a fundamental parameter in any conformal invariant model, and the value c = 1 is of considerable interest, since it forms in many ways a threshold value. For c < 1 a complete classification of all unitary models has been obtained, but c > 1 is still very much terra incognita. Our results give a partial classification for the intermediate case of c = 1 models. The formulation of these c = 1 CFT's on surfaces of arbitrary topology is central in Chapter 7. Here we will provide many explicit results that provide illustrations for our more abstract discussions of higher genus quantities in Chapters 3 and 1. Unfortunately, our calculations will become at this point rather technical, since we have to make extensive use of the mathematics of Riemann surfaces and their coverings. Finally, in Chapter 8 we leave the two-dimensional point of view that we have been so loyal to up to then, and ascend to threedimensions where we meet topological gauge theories. These so-called Chern-Simons theories encode in a very economic way much of the structure of two-dimensional (rational) conformal field theories, and this direction is generally seen to be very promising. We will show in particular how many of our results of Chapter 5 have a natural interpretation in three dimensions.